DTFS

$$x[n] = \sum_{k=\langle N\rangle} C_x[k]\, e^{j2\pi kn/N}$$

$$C_x[k] = \frac{1}{N}\sum_{n=n_0}^{n_0+N-1} x[n]\, e^{-j2\pi kn/N}$$

DFT

$$X[k] = N\, C_x[k]$$

$$\cos\left(\frac{2\pi n}{N}\cdot q\right) \xleftrightarrow{F.S} \frac{1}{2}\left[\delta(k-q) + \delta(k+q)\right]$$

Example: $z[n]$ sinyalinin fourier seri katsayılarını bulunuz.

$$z[n] = x[n]y[n]$$

$$x[n] = \sin\left(\frac{2\pi n}{N} + \pi\right) + 3\cos\left(\frac{4\pi n}{N} + \frac{\pi}{2}\right)$$

$$C_y[k] = \delta[k] + \delta(k-1)$$

Solution:

$$x[n] = x_1[n] + x_2[n]$$

$$x_a[n] = \cancel{\sin\frac{2\pi n}{N}}\ \sin\left(\frac{2\pi n}{N}\right) \xleftrightarrow{F.S} \frac{j}{2}\left[\delta(k+1) - \delta(k-1)\right] e^{-j2\pi k(-\frac{N}{2})/N}$$

$$x_a[n-n_0] = x_1[n] = \sin\left(\frac{2\pi(n-n_0)}{N}\right)$$

$$= \sin\left(\frac{2\pi n}{N} - \underbrace{\frac{2\pi n_0}{N}}_{\pi}\right) \Rightarrow n_0 = -\frac{N}{2}$$

$$\Rightarrow x_1[n] = \frac{j}{2}\left[\delta(k+1)\right]e^{j\pi k} - \frac{j}{2}\left[\delta(k-1)\right]e^{j\pi k} = -\frac{j}{2}\delta(k+1) + \frac{j}{2}\delta(k-1)$$

$$x_b[n] = \cos\left(\frac{4\pi n}{N}\right) = \frac{1}{2}\left[\delta(k-2) + \delta(k+2)\right]\cdot e^{-j2\pi k(-\frac{N}{8})/N}$$

$$\boxed{x[n] * \delta(n-n_0) = x(n-n_0)}$$

$$x_b(n-n_0) = x_2[n] = \cos\left(\frac{4\pi(n-n_0)}{N}\right)$$

$$= \cos\left(\frac{4\pi n}{N} - \frac{4\pi n_0}{N}\right) \rightarrow n_0 = -\frac{N}{8}$$

$$\frac{3}{2}\delta(k-2)e^{j\pi k/4} + \frac{3}{2}\delta(k-2)e^{j\pi k/4}$$

$$\left[-\frac{j}{2}\delta(k+1) + \frac{j}{2}\delta(k-1) + \frac{3j}{2}\delta(k-2) + \frac{3j}{2}\delta(k+2)\right] * \left[\delta(k) + \delta(k-1)\right]$$

$$x[k] * \left[\delta(k) + \delta(k-1)\right] = x[k] + x[k-1]$$

$$C_z[k] = C_x[k] * C_y[k] \Rightarrow$$

Soru: Parseval teoremini kullanarak sinyalin enerjisini bulun.

$$x[n] = \underbrace{\text{sinc}\left(\frac{n}{10}\right)}_{x_1} \underbrace{\sin(2\pi n/4)}_{x_2}$$

$$X(F) = X_1(F) \circledast X_2(F)$$

$X_1(F)$:

$$\text{sinc}(n) \xleftrightarrow{F} \text{rect}(F)$$

$$\text{sinc}\left(\frac{n}{10}\right) \xleftrightarrow{F} 10\,\text{rect}(10F) \circledast \delta_1(F)$$

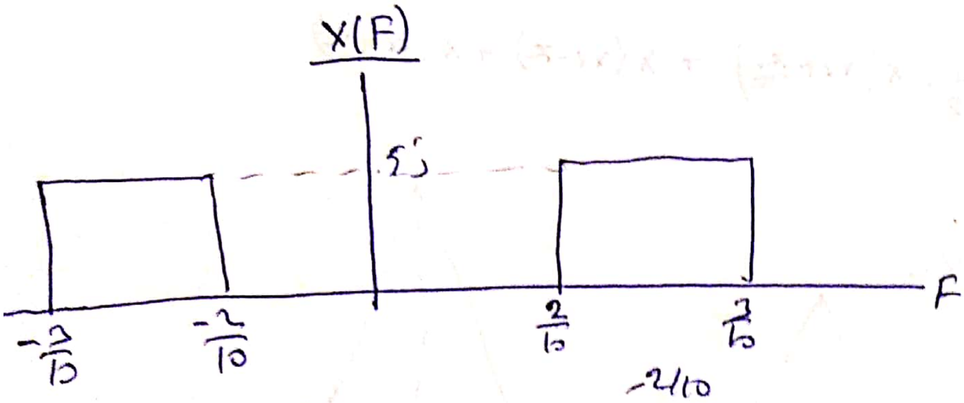
$X_2(F)$:

$$\sin\left(\frac{2\pi n}{4}\right) = \sin(2\pi n f_0) = \frac{j}{2}\left[\delta_1\left(F+\frac{1}{4}\right) - \delta_1\left(F-\frac{1}{4}\right)\right]$$

$$X(F) = X_1(F) \circledast X_2(F) \Longrightarrow \text{1 periyotta konvolasyon}$$

$$= 5j\left[\text{rect}(10F) \circledast \delta\left(f+\frac{1}{4}\right) - \text{rect}(10F) \circledast \delta\left(F-\frac{1}{4}\right)\right]$$

$$= 5j\left[\text{rect}(10F+2{,}5) - \text{rect}(10F-2{,}5)\right]$$



$$\int_{1} |X(F)|^2 df = \int_{-3/10}^{-2/10} |25|\,df + \int_{2/10}^{3/10} |-25|\,df$$

$$= 25\Big|_{-3/10}^{-2/10} + 25\Big|_{2/10}^{3/10} = \boxed{5}$$